The possibility of absolute calibration of analog detectors by using parametric down-conversion: a systematical study

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In this paper we systematically study the possibility of the absolute calibration of analog photodetectors based on the properties of parametric amplifiers. Our results show that such a method can be effectively developed with interesting possible metrological applications.

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I. INTRODUCTION

Accurate calibration of photodetectors both in analog and in photon-counting regime is fundamental for various scientific applications, which range from "traditional" quantum optics [1] to the studies on foundations of quantum mechanics [2], quantum cryptography [3], quantum computation [4], etc.

In traditional optical radiometry primary standards are based on absolute sources or detectors [5]. Synchrotron and blackbody radiations are absolute sources. The spectral radiance of radiation emitted by electron storage rings can be predicted using the Schwinger equation provided that the magnetic field and the electron current have been measured. In a similar way, the radiant flux emitted by a blackbody at a known thermodynamic temperature can be calculated from Planck's equation. Both sources provide radiation with continuous spectral distribution, the dominant contribution being in the soft X-ray and UV spectral region for synchrotron radiation and in the IR spectral region for blackbody radiation. The relative uncertainty of both these sources is about 1 part in 10^3. Among the absolute detectors, there exist the following two types: thermal detector, called electrical substitution radiometer (ESR), and semiconductor photon detector. The ESR is based on the electrical substitution principle, that is, the heating effect of the unknown optical radiation is compared with the heating effect produced by a measured quantity of electrical power (Joule effect). The operation of these detectors at cryogenic temperatures [6], below 20 K, allows to reduce measurement uncertainty down to 1 part in 10^4. The operation principle of semiconductor photon detectors underpins on the photoelectric effect, that is, the generation of free electron-hole pairs at the absorption of a photon. The quantum efficiency is defined as the average number of free electron-hole pairs produced per incident photon. In a high-quality silicon photodiode the quantum efficiency for the visible range is close to unity to within a few tenths of one percent. The deviation of the quantum efficiency from unity could be determined, independently from other radiometric measurements, by self-calibration technique [7]. Uncertainties of a few parts in 10^4 appear to be the limit of this technique with commercial photodiodes. There is a perspective of improvement to 1 part in 10^6 or better with custom photodiodes operated at liquid nitrogen temperatures [8]

For what concerns single-photon detectors, classical calibration techniques are based on the use of a strongly attenuated laser source whose (unattenuated) intensity has been measured by means of a power-meter. The uncertainty of this kind of measurements is principally limited by the uncertainty in the calibration of the very low transmittance required for reaching single-photon level.

This limitation has initiated the study of an alternative scheme, based on the use of photons produced by means of spontaneous parametric down-conversion (SPDC), where photons are emitted in pairs strongly correlated in direction, wavelength and polarization. Furthermore, photons of the same pair are emitted within tens of femtoseconds. Since the observation of a photon of a pair on a certain direction (signal) implies the presence of another one on the conjugated direction (idler), when this last is not observed this occurs because of the non-ideal quantum efficiency of the idler detector, which can be measured in this way [9, 10, 11, 12, 13]. This absolute technique (and others related [14]) is becoming attractive for national metrological institutes to establish absolute radiometric standards because it relies simply on the counting of events, involves a remarkably small number of measured quantities, and does not require any standards.

Because of the success of the SPDC scheme for calibrating single-photon detectors, it is important to analyse if similar absolute calibration methods could be developed for analog ones, eventually allowing the development of a calibration scheme operating in both regimes.

A seminal attempt in this sense was done in [15] following the theoretical proposal of [10]. Nevertheless, these results were limited to the case of very low intensity (as we will show in detail later) and were very far from being developed to the metrological level. An accurate analysis of the possibility to calibrate analog detectors by using SPDC overcoming these limits is therefore demanded.

Incidentally, the quantum efficiency η of analog detectors appears also in equations describing suppression of photon noise in parametric down-conversion using the feedforward [22] or feedback [21],[23] transformations. Hence, such experiments could eventually be used to develop an alternative scheme for analog detector calibra-

tion.

The purpose of this paper is a systematical study of the possibility to calibrate analog detectors by using parametric down conversion.

In section II we will give a short presentation of the SPDC scheme used in the photon-counting regime. In Section III we will analyze multimode SPDC, following a theoretical description of SPDC [16] developed with an account for the high-gain regime (of which we give a short summary). After discussing some general results, we suggest possible calibration methods and show how the scheme of Refs. [10, 15] can be derived as the low-intensity limit of one of them. Finally, in Section IV we will consider the possibility of calibrating analog detectors using the schemes for photocurrent fluctuation suppression. In particular, two-mode squeezing and feedforward techniques will be discussed. For these schemes, we will analyse new possibilities and limitations.

II. THE SPDC SCHEME FOR CALIBRATING SINGLE-PHOTON DETECTORS

The scheme for calibrating single-photon detectors by using SPDC is based on the specific properties of this process, where a photon of the pump beam (usually a laser beam) "decays" inside a non-linear crystal in two lower-frequency photons, 1 and 2 (conventionally dubbed "idler" and "signal"), such that energy and momentum are conserved ($\omega_{pump} = \omega_1 + \omega_2$, $\vec{k}_{pump} = \vec{k}_1 + \vec{k}_2$). Moreover, the two photons are emitted within few femtoseconds. In synthesis, the calibration procedure consists [12] of placing a couple of photon-counting detectors downstream to the nonlinear crystal, along the direction of propagation of correlated photon pairs for a selected pair of frequencies: the detection of an event by one of the two detectors guarantees with certainty, due to the SPDC properties, the presence of a photon with a fixed wavelength on the conjugated direction. If N is the total number of photon pairs emitted from the crystal in a given time interval and $\langle N_1\rangle$, $\langle N_2\rangle$ and $\langle N_c\rangle$ are the mean numbers of events recorded during the same time interval by the signal detector, the idler detector, and in coincidence, respectively, we have the following obvious relationships [10]:

$$\langle N_1\rangle = \eta_1 N;\ \langle N_2\rangle = \eta_2 N, \tag{1}$$

where η_1 and η_2 are the detection efficiencies in the signal and idler arms. The number of coincidences is

$$\langle N_c\rangle = \eta_1\eta_2 N, \tag{2}$$

due to the statistical independence of the two detectors. Then the detection efficiency can be found as

$$\eta_1 = \langle N_c\rangle/\langle N_2\rangle. \tag{3}$$

This simple relation, slightly modified by taking into account the background subtraction and corrections for the acquisition dead-time, is the basis for the scheme for absolute calibration of single-photon detectors by means of SPDC, which reaches now measurement precision competitive with traditional methods [13].

III. ANALOG DETECTION

A. Basic formulas

In order to study the possibility of absolute calibration of analog detectors we are interested in a model of SPDC working at any values of parametric gain. Firstly, the reason is the necessity to work with quite large intensities yielding continuous photocurrent. Secondly, we need to explore new possibilities to use the properties of SPDC for calibration without the usual low-gain limitation. A theory suited for these purposes is developed in [16] and literature cited therein. For simplicity of description we will consider type-I SPDC in the degenerate case where the signal and idler frequencies are $\omega_1 = \omega_2 = \frac{\omega_{pump}}{2}$. In the limit of monochromatic and plane-wave pump approximation, only pairs of modes with opposite transverse wave vectors, $\mathbf{q}$ and $-\mathbf{q}$, and with frequencies $\omega_1 - \Omega$ and $\omega_2 + \Omega$ are coupled as a consequence of energy and transverse momentum conservation. We can write the input-output transformation relating the field operator $a(q,\Omega)$ at the input face of the nonlinear crystal to the field operator $b(q,\Omega)$ at the output face:

$$b(q,\Omega) = u(q,\Omega)a(q,\Omega) + v(q,\Omega)a^\dagger(-q,-\Omega). \tag{4}$$

The coefficients u and v are considered in [17]. For our analysis, we are interested not in the exact form of u and v, but rather in the properties

$$|u(\mathbf{q},\Omega)|^2 - |v(\mathbf{q},\Omega)|^2 = 1, \tag{5}$$
$$u(\mathbf{q},\Omega)v(-\mathbf{q},-\Omega) = u(-\mathbf{q},-\Omega)v(\mathbf{q},\Omega), \tag{6}$$

which guarantee the conservation of the free-field commutation relations $[b(\mathbf{q},\Omega), b^\dagger(\mathbf{q},\Omega)] = \delta(\mathbf{q}-\mathbf{q}')\delta(\Omega-\Omega')$ and $[b(\mathbf{q},\Omega), b(\mathbf{q},\Omega)] = 0$.

The far field observed in the focal plane of a thin lens of focal length f is obtained in [17] from the near field by means of the following transformation:

$$B(\mathbf{x},t) = \frac{-i}{\lambda_s f}\int_{S_A} d\mathbf{x}' b(\mathbf{x}',t) e^{-i\frac{2\pi}{\lambda_s f}\mathbf{x}\cdot\mathbf{x}'}, \tag{7}$$

where $\lambda_s = \frac{2\pi c}{\omega_s}$ is the central free-space wavelength of the down-converted light and S_A is the transverse area of the

domain where PDC takes place. In practical situations it can be identified with the effective cross-section area of the pump beam. We stress that Eq. (7) is correctly usable only for the calculation of normally-ordered correlation functions because it does not conserve the correct commutation relations. According to (4), (7), one can write the far field as

$$B(\mathbf{x},t) = \frac{2\pi i}{\lambda_s f}\int \frac{d\Omega}{\sqrt{2\pi}} e^{-i\Omega t}\int d\mathbf{x}' p(\mathbf{x}-\mathbf{x}')\cdot \tag{8}$$
$$\cdot[\widetilde{u}(\mathbf{x}',\Omega)a(\frac{2\pi}{\lambda_s f}\mathbf{x}',\Omega) + \widetilde{v}(\mathbf{x}',\Omega)a^\dagger(-\frac{2\pi}{\lambda_s f}\mathbf{x}',-\Omega)], \tag{9}$$

where

$$\widetilde{u}(\mathbf{x},\Omega) = u(\frac{2\pi}{\lambda_s f}\mathbf{x},\Omega),$$
$$\widetilde{v}(\mathbf{x},\Omega) = v(\frac{2\pi}{\lambda_s f}\mathbf{x},\Omega). \tag{10}$$

The spatial variation scale of these coefficients is on the order of $x_0 = \sqrt{\frac{\lambda_s f}{2\pi l_c}}$, l_c being the length of the crystal. One can interpret x_0 as the transverse coherence length of SPDC in the focal plane of the lens. The variation scale of $\widetilde{u}$ and $\widetilde{v}$ in frequency, let us denote it Ω_0, represents the typical bandwidth of SPDC in the temporal frequency domain and $\tau_{coh} = \frac{1}{\Omega_0}$ is the coherence time. At the same time,

$$p(\mathbf{x}) = \left(\frac{-i}{\lambda_s f}\right)^2 \int_{S_A} d\mathbf{x}' e^{-i\frac{2\pi}{\lambda_s f}\mathbf{x}\cdot\mathbf{x}'} \tag{11}$$

is the diffraction pattern in the far-field plane due to the finite transverse size of the system. Also here we are not interested in the exact form of $p(\mathbf{x})$, which is determined by the shape of S_A, but we observe that its typical size is $S_{diff} = (\lambda_s f)^2/S_A$ and its amplitude $\frac{1}{S_{diff}}$. Hereafter let us assume the size of S_{diff} to be much smaller than the coherence length of PDC in the far field, i.e., $x_{diff} \ll x_0$. Within this approximation the integrals can be evaluated considering $p(\mathbf{x})$ as a delta function. Now we can calculate the mean value of the photon flux density operator $I(\mathbf{x},t) \equiv B^\dagger(\mathbf{x},t)B(\mathbf{x},t)$ in the detection plane. By using Eqs. (8) and (4), and considering the vacuum as the input state, one obtains

$$\langle I(\mathbf{x},t)\rangle = \frac{1}{S_{diff}}\int d\Omega |\widetilde{v}(\mathbf{x},\Omega)|^2. \tag{12}$$

The physical meaning of this quantity is the mean number of photons crossing the detection plane at point $\mathbf{x}$ at time t per unit area and time. The integral function is usually referred to as the spectral gain of SPDC and its height represents the mean number of photons per coherence time.

We are also interested in the second-order correlation function of the intensity fluctuations defined as

$$\langle \delta I(\mathbf{x},t)\delta I(\mathbf{x}',t')\rangle \equiv \langle I(\mathbf{x},t)I(\mathbf{x}',t')\rangle - \langle I(\mathbf{x},t)\rangle\langle I(\mathbf{x}',t')\rangle, \tag{13}$$

where $\langle I(\mathbf{x},t)I(\mathbf{x}',t')\rangle$ is determined by the joint probability of a photon reaching the detection plane at $\mathbf{x}'$ at time t' and the other one, at $\mathbf{x}$ at time t. It is convenient to distinguish between the two contributions, one due to the autocorrelation inside one beam (signal or idler), the other one due to the cross-correlation between the two beams:

$$\langle \delta I(\mathbf{x},t)\delta I(\mathbf{x}',t')\rangle = \mathcal{G}_{11}(\mathbf{x},t,\mathbf{x}',t') + \mathcal{G}_{12}(\mathbf{x},t,\mathbf{x}',t'). \tag{14}$$

Here $\mathcal{G}_{11}(\mathbf{x},t,\mathbf{x}',t')$ represents the autocorrelation contribution, when the distance between $\mathbf{x}$ and $\mathbf{x}'$ is less than x_{diff}. On the contrary, $\mathcal{G}_{12}(\mathbf{x},t,\mathbf{x}',t')$ is different from zero only if $\mathbf{x}' \simeq -\mathbf{x}$ within x_{diff}. Therefore, it is the term responsible for the well-known cross-correlation between signal and idler beams. From Eqs. (8) and (4), it is easy to show that

$$\mathcal{G}_{11}(\mathbf{x},t,\mathbf{x}',t') = \langle I(\mathbf{x},t)\rangle\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') +$$
$$+|p(\mathbf{x}-\mathbf{x}')|^2\int\int\frac{d\Omega d\Omega'}{2\pi}[e^{-i(\Omega-\Omega')(t'-t)}$$
$$|\widetilde{v}(\mathbf{x},\Omega)|^2|\widetilde{v}(\mathbf{x},\Omega')|^2], \tag{15}$$

$$\mathcal{G}_{12}(\mathbf{x},t,\mathbf{x}',t') = |p(\mathbf{x}+\mathbf{x}')|^2\int\int\frac{d\Omega d\Omega'}{2\pi}e^{-i(\Omega-\Omega')(t'-t)}$$
$$\widetilde{v}^*(\mathbf{x},\Omega)\widetilde{u}^*(-\mathbf{x},-\Omega)\widetilde{v}(\mathbf{x},\Omega')\widetilde{u}(-\mathbf{x},-\Omega'). \tag{16}$$

The first term on the right-hand side of Eq. (15) is due to the commutator $[B(\mathbf{x},t), B^\dagger(\mathbf{x}',t')] = \delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$ and it is responsible for the rise of shot noise during detection, while the second term is the normally ordered auto-correlation. In Eq. (16) the shot-noise term is not present because the commutator is null, being the cross-correlation calculated for $\mathbf{x} \simeq -\mathbf{x}'$. Let us consider two detectors, 1 and 2, having quantum efficiencies η_1 and η_2, to register photons crossing two arbitrary but symmetrically placed regions R_1 and R_2 in the detection plane. Let at least one of the two areas be much bigger than the diffraction area S_{diff} [26]. In fact, from the works on photon-counting detectors calibration it is well known that the area of the detector under investigation should be large enough to collect all the photons correlated with those incident on the trigger detector surface, otherwise the quantum efficiency is underestimated. Integration of Eq. (12) over R_1 gives the photon flux reaching detector 1:

$$\langle I_1\rangle = \frac{1}{S_{diff}}\int_{R_1} d\mathbf{x}\int d\Omega|\widetilde{v}(\mathbf{x},\Omega)|^2 \tag{17}$$

The autocorrelation and cross-correlation functions of the photon fluxes can be obtained by integrating definition (13), respectively, over $R_1 \times R_1$ and $R_1 \times R_2$ and using Eqs. (15) and (16):

$$\langle I_1(t)I_1(t')\rangle = \langle I_1\rangle^2 + \langle I_1\rangle\delta(t-t') + \\ + \frac{1}{S_{diff}}\int_{R_1} d\mathbf{x} \int\int \frac{d\Omega d\Omega'}{2\pi}[e^{-i(\Omega-\Omega')(t'-t)} \\ |\widetilde{v}(\mathbf{x},\Omega)|^2|\widetilde{v}(\mathbf{x},\Omega')|^2], \tag{18}$$

$$\langle I_1(t)I_2(t')\rangle = \langle I_1\rangle\langle I_2\rangle + \frac{1}{S_{diff}}\int_{R_1} d\mathbf{x} \int\int \frac{d\Omega d\Omega'}{2\pi} \\ e^{-i(\Omega-\Omega')(t'-t)}\widetilde{v}^*(\mathbf{x},\Omega)\widetilde{u}^*(-\mathbf{x},-\Omega)\widetilde{v}(\mathbf{x},\Omega')\widetilde{u}(-\mathbf{x},-\Omega'). \tag{19}$$

In the quantum theory approach the shot noise arises from the commutation relations of the free-field operators, see, for example, [19]. Any losses in the channels must be theoretically described, so that the correct commutators are preserved until the moment of the measurement, i.e., the photon-electron conversion inside the detector. Thus, to take into account the losses due to the non-ideal quantum efficiency, we consider any real detector as an ideal one preceded by a beam splitter with the transmission coefficient equal to the quantum efficiency η of the real detector. We substitute the field $B(\mathbf{x},t)$ entering the beam splitter with the transmitted field $C(\mathbf{x},t)$ defined as [1]

$$C(\mathbf{x},t) = \sqrt{\eta}B(\mathbf{x},t) + \sqrt{1-\eta}V(\mathbf{x},t), \tag{20}$$

where $V(\mathbf{x},t)$ is the field operator for the second input port of the beam splitter, which is assumed to be in the vacuum state. From this commutator-preserving transformation, it easily turns out how to regard sub-unity quantum efficiency: it is sufficient to replace, in the normally ordered products, $B(\mathbf{x},t)$ with $\sqrt{\eta}B(\mathbf{x},t)$ [19].

B. Feasibility of analog detectors calibration by measuring SPDC correlations

For our calculations, we will consider a few millimeters non-linear crystal pumped by a CW laser. If the waist of the pump, identifiable with the transverse cross-section of the system S_A, is relatively large, namely on the order of millimeter or more, the real system fits the model of SPDC discussed in the previous section.

In any detectors the absorbtion of a single photon, i.e., any detection event, generates an electric pulse in the current or in the voltage having a profile $f(t)$ and a random area q_n at a random time of occurrence t_n. In the analog process, with a large incident photon flux, we cannot distinguish between two different pulses because they overlap. Rather, the information about the statistics of light is carried by the continuous fluctuations of the photocurrent. We express the current (or voltage) as a superposition of many pulses [20]:

$$i(t) = \sum_n q_n f(t-t_n).$$

In an ideal instantaneous-response photocell, all values q_n are equal to the charge e of a single electron and $f(t) \sim \delta(t)$. For real detectors, the time constant τ_p is finite, and typically $\tau_p \sim$ 1ns or more. Also, when a photodetector is operated in the avalanche multiplication mode, this process gives rise to an internal current gain. The statistical nature of the multiplication process gives an additional contribution to the current fluctuations [25].

Since the probability density of observing a photon at time t is related to the quantum mean value of the photon flux $\langle I(t)\rangle$, we calculate the average current as

$$\langle i_1(t)\rangle = \sum_n \langle q_{1n} f(t-t_{1n})\rangle = \\ \int dt_1 \langle q_1\rangle f(t-t_1)\langle I_1(t)\rangle \, . \tag{21}$$

Analogously, the quantum-mechanical second-order intensity correlation function is determined by the probability density to have a photon detected at time t and another one at time t', see Eqs. (13). Therefore the correlation functions for the current fluctuations can be expressed as

$$\langle i_1(t)i_1(t+\tau)\rangle = \sum_{n,m}\langle q_{1n}q_{1m} f(t-t_{1n})f(t-t_{1m}+\tau)\rangle = \\ \int\int dt_1 dt_1' \langle q_1 q_1'\rangle f(t-t_1)f(t-t_1'+\tau)\langle I_1(t_1)I_1(t_1')\rangle, \tag{22}$$

$$\langle i_1(t)i_2(t+\tau)\rangle = \sum_{n,m}\langle q_{1n}q_{2m} f(t-t_{1n})f(t-t_{2m}+\tau)\rangle = \\ \int\int dt_1 dt_2 \langle q_1 q_2\rangle f(t-t_1)f(t-t_2+\tau)\langle I_1(t_1)I_2(t_2)\rangle, \tag{23}$$

where the first equation is the electron current autocorrelation function for each detector, one registering the intensity of the signal beam and the other registering the intensity of the idler beam, while the second one is the cross-correlation function between the electron currents produced by the two different detectors.

According to Eqs. (21) and (17), the mean value of the electron current (the analog of Eq. (1) in photon-counting regime) is equal to

$$\langle i_1\rangle = \eta_1\langle q_1\rangle\langle I_1\rangle \, . \tag{24}$$

Here the quantum efficiency has been taken into account according to formula (20), with the substitution $\langle I_1\rangle \rightarrow \eta_1\langle I_1\rangle$. The factor $\langle q_1\rangle$ is the average charge produced in a detection event.

The current correlation functions can be obtained substituting Eqs. (18) and (19), respectively, in (22) and (23). We note that functions in the integrals of (18) and (19) have significant values for $|\Omega-\Omega'| \sim \Omega_0$. Roughly, this means that the correlation functions have a sinc-like behavior in time, with the central peak width equal to the coherence time of PDC $\tau_{coh} = \frac{1}{\Omega_0} \sim 10^{-13}$s. As mentioned above, the resolving time of a real analog detector is finite, and in general can be considered much larger than the SPDC coherence time. Thus any fluctuations in the intensity of light are averaged over τ_p during the detection process. So in the limit $\tau_p \gg \tau_{coh}$ we have

$$\langle i_1(t)i_1(t+\tau)\rangle = \langle i_1\rangle^2 + \eta_1\langle q_1^2\rangle\mathcal{F}(\tau)\cdot \cdot\left[\langle I_1\rangle + \eta_1\frac{1}{S_{diff}}\int_{R_1} d\mathbf{x}\int\frac{d\Omega}{2\pi}|\widetilde{v}(\mathbf{x},\Omega)|^4\right] \qquad (25)$$

and (the analog of Eq. (2) in the photon-counting regime)

$$\langle i_1(t)i_2(t+\tau)\rangle = \langle i_1\rangle\langle i_2\rangle + \eta_1\eta_2\langle q_1\rangle\langle q_2\rangle\mathcal{F}(\tau)\cdot \cdot\left[\langle I_1\rangle + \frac{1}{S_{diff}}\int_{R_1} d\mathbf{x}\int\frac{d\Omega}{2\pi}|\widetilde{v}(\mathbf{x},\Omega)|^4\right], \qquad (26)$$

where we have defined

$$\mathcal{F}(\tau) \equiv \int dt f(t)f(t+\tau). \qquad (27)$$

The last two equations are the fundamental tools for studying the problem of absolute calibration of analog detectors and thus we are going to discuss them in detail. Despite (25) and (26) seem to be quite symmetric, we observe some important differences. The presence of $\langle I_1\rangle$ in the autocorrelation function is due to the shot noise contribution and for this reason the quantum efficiency η enters linearly, while in the current cross-correlation function the corresponding term is due to the high quantum correlation between the signal and idler beams of PDC and the quantum efficiency appears quadratically. It is equivalent to the right-hand side of Eq. (2) for the counting regime and its presence is the key for absolute calibration. The second term inside the brackets, both for auto- and cross-correlation functions, is important only when the number of photons per coherence time is not close to zero and so the presence of two or more photons within that time is not negligible. In fact, the integral term can be estimated as $v^2\langle I_1\rangle$ and can be neglected as long as $v^2 \ll 1$, i.e., the mean number of photons per τ_{coh} is much smaller than one. Anyway, if the duration of the photocurrent pulse is much larger than the coherence time, this assumption does not prevent photodetection to be in a strongly analog regime, because a lot of photons can be absorbed during the pulse duration as well. The term proportional to $\langle I\rangle^2$ is due to the presence of more than one photon in time τ_p and for that reason is more delicate. As we can observe in Eq. (26) it can be neglected only if $\langle I\rangle \ll \mathcal{F}(\tau)$. Since the pulse $f(t)$ has a height around $1/\tau_p$, by Eq. (27), $max\,[\mathcal{F}(\tau)] = \mathcal{F}(0) \sim 1/\tau_p$. So the condition becomes $\langle I\rangle\tau_p \ll 1$, i.e., the number of incident photons during the resolving time of the detector should be much less than one, i.e. one should work in a non-overlapping regime. In principle, in this case one could distinguish between different pulses of the current and work in the counting mode.

The usual definition of the quantum efficiency is the ratio between the number of photons detected and the number of photons incident on the detector surface. This definition is completely suitable in the case of counting detectors and is exactly the meaning of η in our paper. But in the case of analog detection, we cannot in principle distinguish between different current pulses. Thus, according to formula (24), we adopt the definition of analog quantum efficiency as $\Gamma \equiv \eta\langle q\rangle = \langle i\rangle/\langle I\rangle$, having the meaning of the electron charge produced per single incident photon, or the ratio between the electron flux and the photon flux. If the charge q produced per photon fluctuates it increases the current fluctuations. This explains why $\langle q_1^2\rangle$ appears in Eq. (25) instead of $\langle q_1\rangle^2$. In principle, the most general way to obtain an estimation of η working with the PDC light intensity in the photon-counting regime is dividing the coincidence counting rate (proportional to the cross-correlation function) by the detector counting rate (proportional to the intensity). This method works because in the photon-counting regime the temporal shape of the current pulses and their width is not important; instead, one registers a single pulse or not registers it. This is not the case for analog detection in which the pulse shape $f(t)$ appears in formulas through the factor $\mathcal{F}(\tau)$. In general, we do not know this function, and this makes the absolute calibration of analog detectors more difficult. However, as we are going to show, under some condition it is possible to overcome this drawback.

Let us distinguish between three different regimes: very low intensity (I), middle intensity (II), high intensity (III).

(I) $\langle I\rangle\tau_p \ll 1$ (i.e., photocurrent pulses do not overlap). For example, for a detector with a time constant $\tau_p = 10$ ns, the corresponding photon flux must be below 10^8 photons/s. In terms of power, for the wavelength of 500 nm, it means about 10 pW.

Eqs. (25) and (26) become then

$$\langle i_1(t)i_1(t+\tau)\rangle = \eta_1\langle q_1^2\rangle\mathcal{F}(\tau)\langle I_1\rangle, \qquad (28)$$

$$\langle i_1(t)i_2(t+\tau)\rangle = \eta_1\eta_2\langle q_1\rangle\langle q_2\rangle\mathcal{F}(\tau)\langle I_1\rangle. \qquad (29)$$

The same equations has been found in [10] and the

quantum efficiency has been estimated as

$$\Gamma_2 = \eta_2 \langle q_2 \rangle = \frac{\langle q_1^2 \rangle}{\langle q_1 \rangle^2} \langle q_1 \rangle \frac{\langle i_1(t) i_2(t+\tau) \rangle}{\langle i_1(t) i_1(t+\tau) \rangle}. \qquad (30)$$

This formula is not satisfying from the metrological point of view because of the presence of some unknown parameter related to the statistics of charge fluctuations that we discussed previously and that has to be estimated in some other way. We suggest to avoid the problem by integrating Eq. (29) over time τ. It could be done after the acquisition of the profile of the function has been performed. By definition (27), it is evident that $\int d\tau \mathcal{F}(\tau) = 1$; integrating Eq. (29) in τ and dividing it by Eq. (24), we obtain

$$\Gamma_2 = \eta_2 \langle q_2 \rangle = \frac{\int d\tau \langle i_1(t) i_2(t+\tau) \rangle}{\langle i_1 \rangle} \qquad (31)$$

As pointed out, another drawback of (30) is the necessity to work only at very low intensity, where no overlapping between pulses happens. This, in terms of experiment, means that one has to work in the so-called "charge accumulation mode", in which the electron charge is accumulated until reaching some detectable threshold. In the case of avalanche devices, it corresponds to the possibility of direct photon-counting.

(II) $\langle I \rangle \tau_p \gtrsim 1$ but still $v^2 \ll 1$ (i.e., photocurrent pulses overlap but the parametric gain and photon flux are still quite low). Considering the same parameters as used in case I, coherence time τ_{coh} of the order of 100 fs, and the requirement that $v^2 \leq 0.001$, this means a photon flux between 10^8 photons/s and 10^10 photons/s or power between 10 pW and 100 nW.

Eqs. (25) and (26) become

$$\langle i_1(t) i_1(t+\tau) \rangle = \langle i_1 \rangle^2 + \eta_1 \langle q_1^2 \rangle \mathcal{F}(\tau) \langle I_1 \rangle, \qquad (32)$$

$$\langle i_1(t) i_2(t+\tau) \rangle = \langle i_1 \rangle \langle i_2 \rangle + \eta_1 \eta_2 \langle q_1 \rangle \langle q_2 \rangle \mathcal{F}(\tau) \langle I_1 \rangle. \quad (33)$$

We stress that in [10], to the best of our knowledge the only theoretical paper treating the absolute calibration of analog detectors using PDC, the very low parametric gain was assumed from the very beginning. In particular, all terms proportional to v^4 were neglected. This corresponds to the approximation (I) in our treatment. Accordingly, in [10] the limitations of Eq. (30) were not discussed. Our analysis shows that when the intensity is high enough to yield a strongly analog current, Eqs. (32) and (33) should be used. Now we define the correlation functions of the current fluctuations as

$$\begin{aligned} \langle \delta i_k(t) \delta i_l(t+\tau) \rangle \;\; &\equiv \;\; \langle i_k(t) i_l(t+\tau) \rangle - \\ &- \;\; \langle i_k(t) \rangle \langle i_l(t+\tau) \rangle \quad (k,l = 1,2)\,. \end{aligned} \quad (34)$$

We underline that these functions remain in principle experimentally estimable. So a new formula, similar to (30), is available for analog quantum efficiency estimation:

$$\Gamma_2 = \eta_2 \langle q_2 \rangle = \frac{\langle q_1^2 \rangle}{\langle q_1 \rangle^2} \langle q_1 \rangle \frac{\langle \delta i_1(t) \delta i_2(t+\tau) \rangle}{\langle \delta i_1(t) \delta i_1(t+\tau) \rangle}. \qquad (35)$$

Once again, the drawback of this formula is the presence of the unknown parameter $\langle q_1^2 \rangle / \langle q_1 \rangle^2$ that requires additional measurements to be performed. As before, integrating in τ the expression for the cross-correlation, i.e., the definition (34) for $k = 1, l = 2$, we obtain

$$\Gamma_2 = \eta_2 \langle q_2 \rangle = \frac{\int d\tau \langle \delta i_1(t) \delta i_2(t+\tau) \rangle}{\langle i_1 \rangle}\,. \qquad (36)$$

This equation represents one of the main results of the paper, since it shows that the absolute calibration of analog detectors by using SPDC is indeed possible.

A drawback of quantum efficiency measurement in this regime could derive from the fact that the terms proportional to the square of intensity in (32) and (33) become rather large, much larger than the term proportional to the intensity - the one that provides calibration. We need to subtract this background, as it is done in (34). Although it is an easily estimable quantity, a little relative uncertainty could generate a large uncertainty in the efficiency estimation: a limitative result when looking toward metrological applications. The physical reason for this behaviour can be found recalling that the quantum correlation so attractive in PDC has the scale of τ_{coh}. In the analog regime this correlation is almost deleted because of the averaging over a time $\tau_p \gg \tau_{coh}$. Anyway, as long as we take $\langle I \rangle \tau_p \sim 1 \div 10$, such a problem does not arise.

An alternative could be working in the pulsed regime, in which the duration of any pump pulses is not so far from the coherence time of PDC and the distance between them is larger than τ_p. Of course, to have a large number of photons during τ_p (strongly overlapping regime) we need to increase the parametric gain. A detailed study of this possibility will be presented elsewhere.

(III) $v^2 \gtrsim 1$ (i.e., high-intensity regime).

In this regime each term of (25) and (26) is important and no general way can be found for the absolute calibration of analog detectors, at least with a CW pump. It can be shown that in the single-mode case the integral terms in (25) and (26) become equal to the first ones, proportional to the square intensity, and thus could be easily estimated. In the case of CW pump, single-mode detection requires very narrow filters and fast detectors, beyond realistic present technological possibilities. Furthermore, calculation shows that for analog calibration we need to know the transmission spectrum of the filters. Anyway, also in this case one can consider the possibility to reach single-mode detection by using a pulsed pump.

IV. SQUEEZING FOR CALIBRATION

In the 1980's and the 1990's a lot of work has been done to demonstrate both theoretically and experimentally the possibility to obtain sub-shot-noise photocurrent statistics taking advantage of strong PDC quantum correlation. The shot-noise level (SNL) is defined to be the lower limit to the photocurrent noise level, which is achieved for coherent states of the field. Basically, for what concerns two-mode squeezing, two different kinds of schemes have been used. The first one consists of detecting the currents from the two light beams (signal and idler) and subtracting them [24]. The variance of the difference current or the difference between the numbers of generated electrons can be less than the same quantity measured for coherent beams. In the other scheme the information about fluctuations in one beam is used to manipulate the intensity of the second beam (feedforward technique) [22], or directly the pump intensity (feedback technique) [23] in order to correct the fluctuations. The goal in this case is getting a reduction of photocurrent fluctuations below the shot-noise level for the detector measuring one of the beams.

It is important that in both schemes the minimum reachable squeezing factor depends strongly on the quantum efficiency of the detectors. Thus, it is reasonable to consider the possibility to extract the quantum efficiency from the degree of squeezing.

Let us first consider the two-mode squeezing introducing the difference between the currents $i_- = i_1 - i_2$. Using equations (25) and (26) the autocorrelation function can be easily evaluated. When we consider two balanced detectors, $\eta_1 \langle q_1 \rangle = \eta_2 \langle q_2 \rangle = \eta \langle q \rangle$, collecting photons from symmetric and equal detection areas, and no fluctuations in the charge produced per photon occur, i.e., $\langle q^2 \rangle = \langle q \rangle^2$, we have

$$\langle \delta i_-(t) \delta i_-(t+\tau) \rangle = SNL(1-\eta), \qquad (37)$$

where the shot noise level is given by SNL $= 2\langle q \rangle^2 \eta \langle I \rangle \mathcal{F}(\tau)$. Let us discuss this formula. First of all, its validity does not disappear in the high-gain regime and at high light intensity because the integral terms of (25) and (26) cancel each other in the calculation of (37). Quite surprisingly, it shows that some aspects of the quantum correlation, such as the possibility of shot-noise suppression, can be preserved when working with macroscopic intensities, where the cross-correlation is dominated by classical terms proportional to the square of the intensity. Furthermore, it opens the possibility of calibrating analog detectors with SPDC at high parametric gain. In particular, it can be useful in the calibration of detectors in which the electronic noise of the external amplifier is dominant compared to the shot noise when working at low intensities (cases (I) and (II) of the previous section). In this kind of detectors, like, for instance, simple photodiodes, no avalanche or multiplication occurs and in general we can consider the condition $\langle q^2 \rangle = \langle q \rangle^2$ satisfied. So the requirements for the derivation of (37) are realistic in this case. It can be used for calibration directly, by calibrating the SNL with the help of a coherent source producing the same average current as measured for one of the SPDC beams, or by integrating in τ and then normalizing by $\langle i_1 \rangle$:

$$\frac{\int d\tau \langle \delta i_-(t) \delta i_-(t+\tau) \rangle}{\langle i_1 \rangle} = 2\langle q \rangle (1-\eta). \qquad (38)$$

From the experimental viewpoint, the validity of (37) is guaranteed if the detection volumes of the two detectors are conjugated. In other words, detector 2 has to collect exactly all and only the modes conjugated with those collected by detector 1. For that pertaining the spatial modes, detailed realistic calculation performed in [16] as well as recent experiments [18] show that the detection areas should be large enough to collect several spatial modes. By satisfying this condition, one would also increase the detected radiation power and hence, the signal-to-noise ratio.

Finally, we note that an equivalent formula is valid for the counting regime without any assumptions except $\eta_1 = \eta_2 = \eta$:

$$\frac{\langle (\delta N_-)^2 \rangle}{\langle N \rangle} = 2(1-\eta), \qquad (39)$$

where $N_- = N_1 - N_2$ is the difference between the numbers of counts in detectors 1 and 2, and $\langle (\delta N_-)^2 \rangle$ is its variance. This method of calibration could be interesting because, unlike the method of coincidence counting based on Eq. (3), it only requires counting photodetection pulses.

Of course the assumption of balanced quantum efficiencies used for deriving (37) somehow complicates the aim of reaching the accuracy needed for metrological applications, although the technique of balancing detectors is largely used in quantum optics.

Finally, we consider the possibility to exploit the feedback or feedforward schemes. In Ref [22], the latter is studied in detail for optical parametric oscillator (OPO) above threshold where one has bright average intensity component and small fluctuations. It is shown that the minimum of fluctuations achievable can be expressed in terms of the frequency spectra of the fluctuations in each beam and the correlation spectrum in the absence of the feedforward action. Unfortunately the results of [22] can not be applied directly to SPDC because in the case of SPDC the resulting state is squeezed vacuum where high fluctuations of intensity occur. Anyway, description of the feedforward scheme presented in [22] can be applied to the currents. For instance let the current in the detector registering beam 1, i_1, be varied by using a modulation mechanism conditioned by the measurement performed on beam 2. If $\widetilde{i_1}$ is the current registered after modulation, in our time representation (for simplicity

with $\tau = 0$) the result of Ref. [22] yields

$$\langle (\delta \widetilde{i_1})^2 \rangle = \langle (\delta i_1)^2 \rangle - \frac{\langle \delta i_1 \delta i_2 \rangle^2}{\langle (\delta i_2)^2 \rangle} . \tag{40}$$

This formula shows that is possible to measure the reduction in the current fluctuations of detector 2 after modulation instead of measuring the cross-correlation function $\langle \delta i_1 \delta i_2 \rangle$. Substituting (32) and (33) into (40), we have

$$\frac{\langle (\delta \widetilde{i_1})^2 \rangle}{\langle (\delta i_1)^2 \rangle} = 1 - \eta_1 \eta_2 \frac{\langle q_1 \rangle^2 \langle q_2 \rangle^2}{\langle q_1^2 \rangle \langle q_2^2 \rangle} . \tag{41}$$

If the two detectors have balanced quantum efficiencies and excess charge noise is absent, the above formula can be used for the efficiency estimation, representing a further interesting option in this sense.

It is interesting to observe that in [23], the authors obtained the same suppression of fluctuations as given by (41) using a theoretical model describing a feedback procedure applied to SPDC. The only difference is that our equation takes into account the statistics of the charge, which enters through the factor $\frac{\langle q_1 \rangle^2 \langle q_2 \rangle^2}{\langle q_1^2 \rangle \langle q_2^2 \rangle}$. Even if the quantum efficiency is ideal, this factor prevents one from reaching perfect noise reduction.

V. CONCLUSION

Motivated by the necessity of a general absolute calibration scheme for analog detectors for various applications and aiming to extend the absolute calibration scheme from the single-photon to the analog regime, we have performed a systematical study on the possibility of applying SPDC calibration methods to the analog regime. Possibilities and limitations following from the specific properties of SPDC have been investigated.

Our results show that measurement of the correlations in the output currents indeed can be used to extend the absolute calibration method to the analog regime, although the experimental implementation will require an accurate study and solution of some technical problems.

In particular, it is shown that integration of the photocurrent correlation functions in time allows one to avoid the measurement of the photocurrent pulse shape and to eliminate the necessity to know the statistics of electrons in the photocurrent.

Also, our analysis showed the possibility to go beyond the regime of non-overlapping photocurrent pulses, which was used in earlier works, and to pass to higher intensities. A possible way to reduce the background in the measured photocurrent correlation function, which will unavoidably accompany the transition to higher intensities, namely, passing to the pulsed regime of PDC, is outlined.

Finally, we studied the new subject of squeezing as a tool for absolute photodetectors calibration. In particular, two-mode squeezing is shown to be the only way of performing calibration in the high-intensity regime of SPDC, where other methods fail, but with the limitation that one should have two detectors with balanced quantum efficiencies. Also, a possibility of estimating the quantum efficiency for the feedforward or feedback scheme of fluctuations suppression is considered, reaching analogous conclusions.

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